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Wavering equation for free recharge in a circumscribed aquifer

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Abstract

Slug test solutions require instantaneous water column in the recharge well, and it consider well storage and well loss not appropriate for recharge cases. State of the art suggests recharge hydraulics, a phenomenon synonymous to mirror image of well pumping. In this background, the present paper derives fresh semi-analytical equations for confined aquifer, which simultaneously determine unsteady recharge rates at the recharge well face and rising heads in the aquifer. Such computations that are mandatory in free recharge situations may not be possible with slug test solutions. Developed solutions in a fully penetrating well include well storage, a function of aquifer diffusivity. Head loss computation is found more appropriate with friction parameter "k", a function of Reynolds number.

Key words: Fully penetrating well, well storage, discrete kernel, friction parameter.

INTRODUCTION

In using free recharge technique, water is injected in to the aquifer maintaining either constant or variable head in the injection well (Sevee, 2006). Recharge rates under constant head can be computed using the method of Jacob and Lohman (1952), whereas variable head conditions are dealt with in slug theories (ASTM, 2001, 2002a, 2004b). Slug test essentially consists of measuring the recovery of head in a well after a near-instantaneous change of head in that well (ASTM, 2002b, 2004a). Response data is mostly analysed using the method of Hvorslev (1951) and Cooper et al. (1967). Papadopulos et al. (1973) extended the work of Cooper et al. (1967) to the aquifers with very low storage coefficients. Bouwer (1989) equation requires a large depth between the top of the screen or open section of the well and the upper confining layer. Pouring water quickly into the wells for generating considerable well storage is practiced in recent years under artificial recharging techniques especially in low permeability aquifers. Under such circumstances, recharge from the well takes place for longer time periods. Cooper et al. (1967) suits extremely short durational recharge process as slug introduced needs to be instantaneous.

McElwee (1994) performed sensitivity analysis of the parameters in Cooper et al. (1967) for slug tests in confined aquifers. Sensitivity of S was found much lower than T and sensitivity curves for these two parameters looked very much similar in shape. Chapuis (1998) also found out that S has negligible influence in the slug test solutions including Cooper et al. (1967). These findings are indicative of improper representation of storativity in the existing solutions, when it is being used for recharge

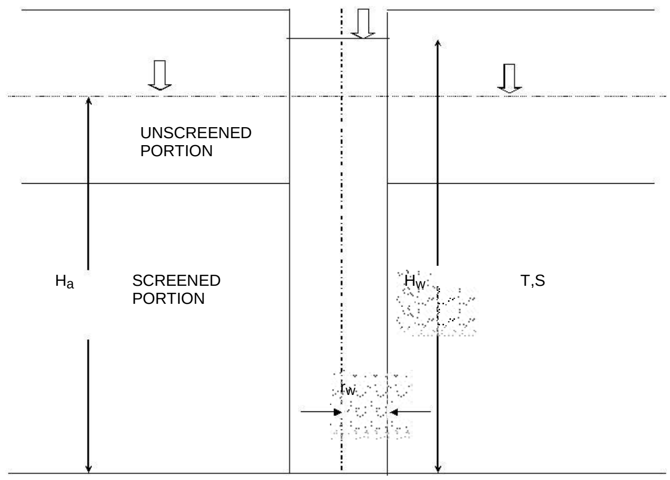

Figure 1. Schematic diagram of a recharge well.

phases. Incidentally, most of the slug solutions consider well storage a fraction of storativity. Moreover, McElwee (2002) found hydraulic conductivity a more sensitive parameter in comparison to slug initiation velocity and slug height, indicating poor influence of well storage. Elsewhere, Peres (1989), Peres et al. (1989) and Spane and Wurstner (1993) compared the effects of well bore storage during slug test and constant rate pumping (or injection as mirror image). Comparison between the two was found meaningful for small initial times only, depending on the diffusivity of the aquifer. In later times, there was no match while well storage became active in pumping test. As well storage plays dominant role during a recharge cycle, pumping test solutions with adequate well storage may suit better for recharge test analysis. Majumdar et al. (2009) developed such solution for forced recharge cases, where well storage conditions are explicitly coupled using discharge terms.

Well loss is generally considered to represent turbulence inside the well and nonlinear head losses at the well screen and within the aquifer. A rate dependant skin component is commonly used to describe the total loss (Jacob, 1947, 1950). Kabala et al. (1985) found out that the impact of nonlinear terms become significant when the initial water level displacement is a large fraction of the effective water column height. Kipp (1985) developed numerical solution taking into account well-bore storage and inertial effects of the water column in the well. Guenther and Mohamed (1986) described the relation between the inertial forces and viscous effect of water within the well column. Guenther et al. (1987) described a numerical model, which takes into account the inertial and friction effects. Zenner (2002) developed a general non-linear model for bypassed wells, including skin effects, non-head losses due to internal well bore fluid friction, minor losses originating at radius changes along the flow path inside the well, and inertial effects of the water columns contained within the primary casing and the bypass. Chen and Wu (2006) mentioned that inertial and frictional forces cause oscillation in the well water level. Such head losses due to inertial and frictional forces during explicit introduction of well storage (Majumdar et al., 2009) using Diritchlet type boundary condition are examined in the present paper.

FREE RECHARGE EQUATION

In Figure 1, schematic cross-section of a recharge well in a single confined aquifer is shown with initial piezometric head in the aquifer at a height H_a from the bottom of the aquifer. Transmissivity and storage coefficient of the aquifer are T and S, respectively. The well screen and unscreened portions have the same radius equals to r_w. To recharge the aquifer, water is stored in the well so as to increase the head from H_a to a height H_w from the bottom of the aquifer. Equation for estimating recharge under unsteady state condition has been presented here. Due to recharge, piezometric water level in the aquifer will go up, whereas, water level in the well will start receding. The piezometric head in the aquifer H_a at radial distance 'r' from the recharge well, at time step 'n' is given by,

$$H_a(r,n) = H_a(r,o) + s(r,n) \qquad (1)$$

Here, s(r, n) is the head rise in the aquifer at a radial distance r from the recharge well. This is obtained by Duhamel's convolution theory, which states that if the recharge to the aquifer (perturbation) can be assumed as a train of pulses, each pulse being constant within a time step, but varying from step to step, then the rise in head

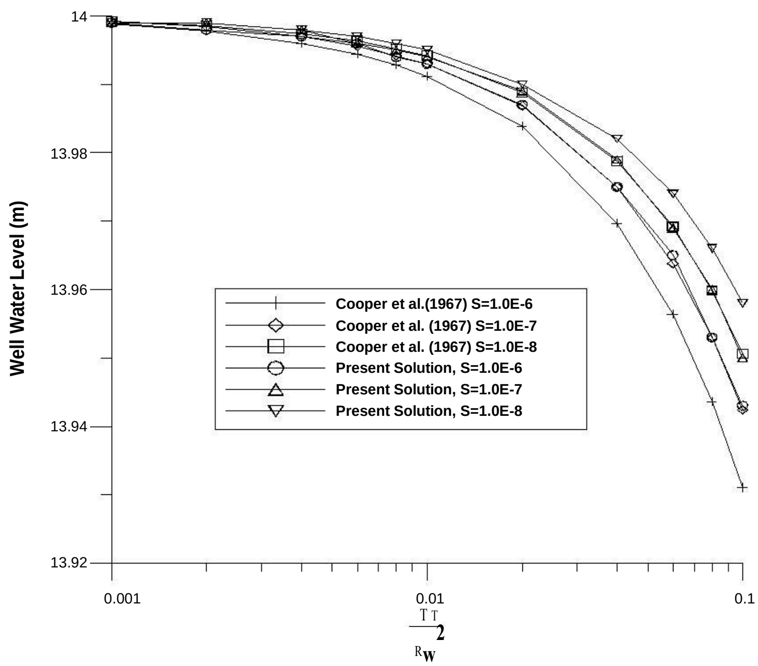

Figure 2. Comparison of the present solution with Cooper et al. (1967).

after the n^{th} time step at distance r is given by (Morel-Seytoux and Daly, 1975):

$$s(r,n)=\sum_{\gamma=1}^{n} Q_a(\gamma)\delta(r,t,n-\gamma+1) \qquad (2)$$

Where δ( ) is the discrete kernel coefficient t in m/m^3, Q_a is the volume of water in m^3 recharged in unit time step at time γ, and Δt is the unit time step.

$\delta(r, t, n-\gamma+1)$ is known as discrete kernel coefficient or delta function, which is the unit response of the system. This is estimated separately by Theis (1935) well function equation for unsteady flow in a confined aquifer. Derivation of delta function is given in Majumdar et al. (2009). From Equations 1 and 2,

$$H_a(r,n)=H_a(r,o)+\sum_{\gamma=1}^{n}Q_a(\gamma)\delta(r, t,n-\gamma+1) \qquad (3)$$

With Δt being unity, receding water table in the well can be expressed as:

$$H_w(n)=H_w(o)-\frac{1}{\pi r_w^2}\sum_{\gamma=1}^{n}Q_a(\gamma) \qquad (4)$$

Majumdar et al. (2009) explains the significance of the unit time step. Assuming no head loss, at r = r_w, $H_a(n)$ would approach $H_w(n)$ as 'n' progresses, and for γ convoluting n number of pulses, $H_a(n) = H_w(n)$. Therefore, equating Equations 3 and 4, thereafter extracting the n_{th} term to left hand side and simplifying,

$$Q_a(n)=\frac{H_w(o)-H_a(o)-\frac{1}{\pi r_w^2}\sum_{\gamma=1}^{n-1}Q_a(\gamma)-\sum_{\gamma=1}^{n-1}Q_a(\gamma)\delta(r, t,n-\gamma+1)}{\frac{1}{\pi r_w^2}+\delta(r, t,1)} \qquad (5)$$

In particular for n=1,

$$Q_a(1)=\frac{H_w(o)-H_a(o)}{\frac{1}{\pi r_w^2}+\delta(r, t,1)} \qquad (6)$$

Discrete kernels are generated for known values of T and S, using the expression for delta function (Majumdar et al., 2009). Using these discrete kernels, values of Q_a are evaluated each time step in succession, making use of Equation 5 for a particular radius of the well casing. In each time step, piezometric water level is calculated using Equation 3 making use of the Q_a value for the previous time step. Finally, $\sum_{\gamma=1}^{\infty}Q_a(\gamma)$ SHOULD NOT BE MORE THAN $\pi r_w^2\{H_w(o)-H_a(o)\}$.

WELL STORAGE DURING FREE RECHARGE

Well water levels obtained using the present equation are compared with the results of Cooper et al. (1967) in Figure 2 for T=1.0E-5 m^2/unit time step and rw=0.1 m.

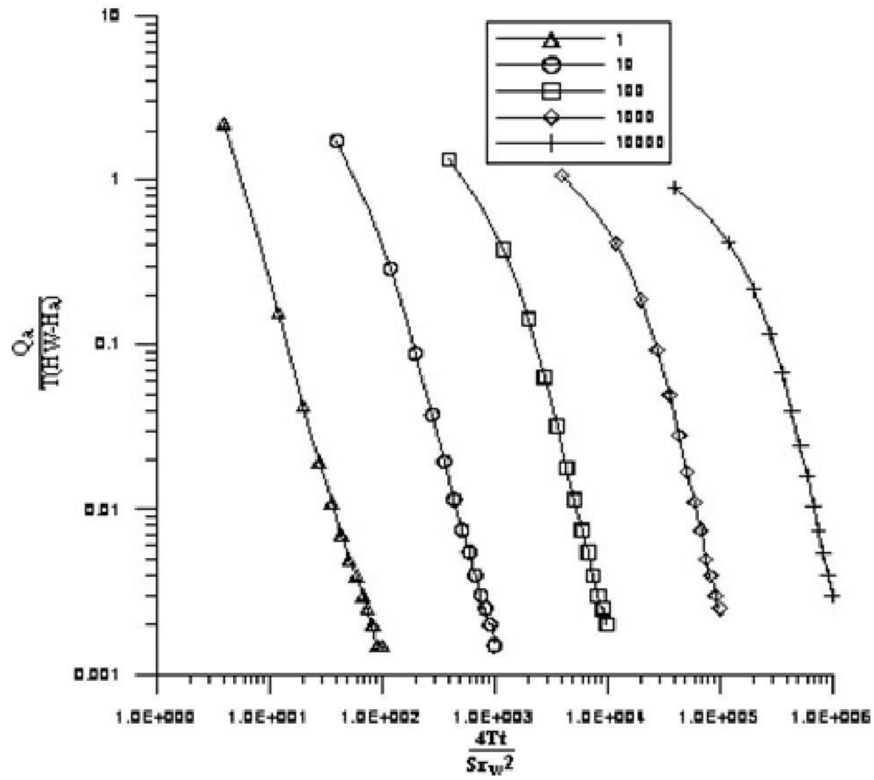

Figure 3. Effect of diffusivity (m^2/day) on recharge rate in a single aquifer.

The present comparisons are purposefully carried out for the equal well radius at screen and casing. Accordingly, well storage in Cooper et al. (1967) becomes equal to S for the entire recharge cycle. It is observed that well water levels estimated by the present equation are higher than those estimated by Cooper et al. (1967) and the difference in the corresponding head values increases with time. Results in Figure 2 also show that the deviation between the two equations increases with the increase in S. This deviation pattern is due to the fact that in the present equation, well storage is time dependant, a function of aquifer diffusivity and recharge well water column height, whereas Cooper et al. (1967) argued on constant well storage, a fraction of S. This is an improved formulation of well storage in a recharge well.

Effects of aquifer parameters, as shown in Figure 3 in non-dimensional form indicate that with aquifer diffusivity, recharge rate shows a near linear behaviour. For high diffusivity, the recharge rate is initially non-linear and tends to become linear in the later times. The exponential nature of the recharge curve is controlled by the transient well storage very similar to the S effects in slug solutions. These are the type curves for the specific values of aquifer diffusivity and well radius. In free recharge problems, where recharge rate decreases with time, decay time is a useful parameter. Decay time presented in Figure 4 is the time when recharge rate tends to become steady. Decay of recharge rate occurs early with the increase in diffusivity. The decay time (t_d) can be estimated with known aquifer and well parameters from Figure 4. This is the useful information available in advance about the likely duration of a proposed recharge test. Inversely, diffusivity of the aquifer can also be estimated with a single field test observation, that is, time to decay of a recharge column in a well of known dimension.

HEAD LOSS IN FREE RECHARGE

Using the notations given in Figure 1 and applying Bernoulli's equation in the recharge well face, between the top levels of recharge column and aquifer,

$$H_a(o) + \sum_{\gamma=1}^{n} Q_a(\gamma)\delta(r, t, n-\gamma+1) = H_w(o) - \frac{1}{\pi r_w^2}\sum_{\gamma=1}^{n} Q_a(\gamma) - \left(\frac{Q_a(n)}{\pi r_w^2}\right)^2 \frac{1+kl}{2g} \quad (7)$$

$$H_w(o) - H_a(o) = \frac{1}{\pi r_w^2}\sum_{\gamma=1}^{n} Q_a(\gamma) + \sum_{\gamma=1}^{n} Q_a(\gamma)\delta(r, t, n-\gamma+1) + \left(\frac{Q_a(n)}{\pi r_w^2}\right)^2 \frac{1+kl}{2g} \quad (8)$$

where k is the friction parameter (f/D) and l is the distance between water level in the well and top of the aquifer, f is the friction factor in Darcy-Weisbach equation

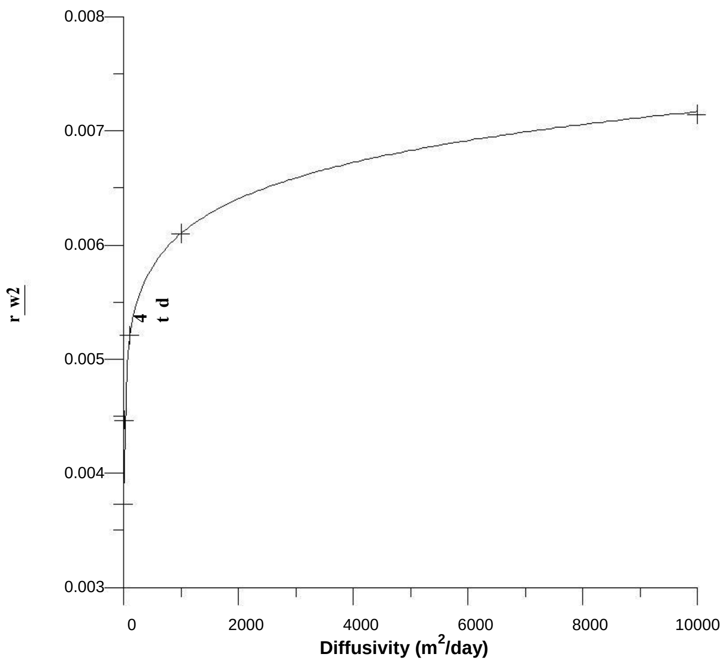

Figure 4. Decay of recharge rates in single aquifer with diffusivity.

for head loss due to friction h_f in the recharge well, which is given by

$$h_f = \frac{flv^2}{2gD} \tag{9}$$

where v is the magnitude of Darcy flux vector and D is the well bore diameter. Friction factor (f) is assigned as per Reynolds number, $R_e = \frac{vD}{\nu}$, where ν is the coefficient of viscosity. Value of friction factor could be extrapolated from Moody's diagram (Featherstone and Nalluri, 1982) for known Reynolds number.

Hence, (10)

$$\frac{Q_a(n)^2}{\pi r_w^2}\frac{1+kl}{2g} + \frac{1}{\pi r_w^2} Q_a(n) + Q_a(n).\delta(r, t, 1) =$$
$$H_w(o) - H_a(o) - \frac{1}{\pi r_w^2}\sum_{\gamma=1}^{n-1} Q_a(\gamma) - \sum_{\gamma=1}^{n-1} Q_a(\gamma)\delta(r, t, n-\gamma+1)$$

$$2\{H_w(o) - H_a(o)\}$$

This is a quadratic equation to be solved for $Q_a(n)$.

$$\sum_{r=1}^{\infty} Q_a(\gamma) \rightarrow \pi r_w \qquad For\ n \rightarrow \infty$$

Here, k is the constant for a specific well. Using discrete kernels, values of Q_a are evaluated each time steps in succession, making use of Equation 10 for a particular radius of the well. Head decline in the well is plotted in Figure 5 for different values of k. It is evident that with increase in k value, the time taken to reach steady water level in the well increases, that is, the decay time increases with increase in k and the recharge rate decreases. Again, the model results are similar to those of slug solutions. The recharge rate to the aquifer is as shown in Figure 6. The values of 'k' chosen for different trials pertain to the 'Re' values that could cover the ranges of laminar as well as turbulent flow in the recharge well.

FIELD APPLICATION

A test problem of Table 1, using the hydro-geological conditions of Hansol case study is considered, and is as shown in Figure 7.

Here, the objective is to test the comparative effectiveness of C_W and k in a field situation. The data analysed pertain to a well recharge experiment conducted around Hansol, near Ahmedabad (Desai et al., 1977). During the experiment, water from a source well was poured into a 238 m deep tube well. Confined aquifer below 74 m depth is recharged during injection with screen portion of the aquifer lying between 74 and 93 m below ground level. Prior to injection, pumping and recharge test data were analysed by more than one method and the average of these methods have been considered as possible aquifer parameters for the present analysis. Single confined aquifer configuration as shown in Figure 8 is based on the injection well litho-logical information and is used for the present analysis.

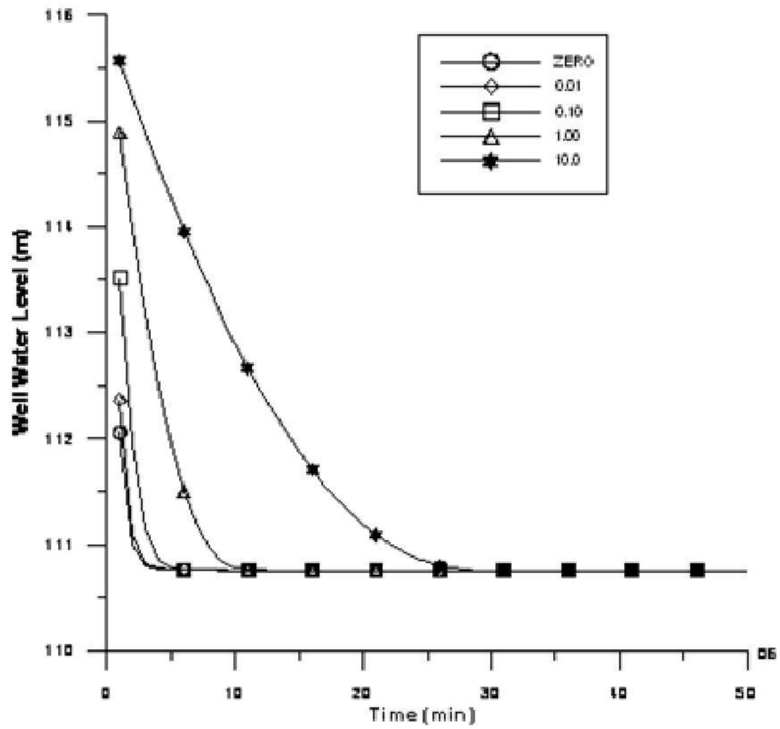

Figure 5. Change in well water level with different friction loss, k (per unit diameter).

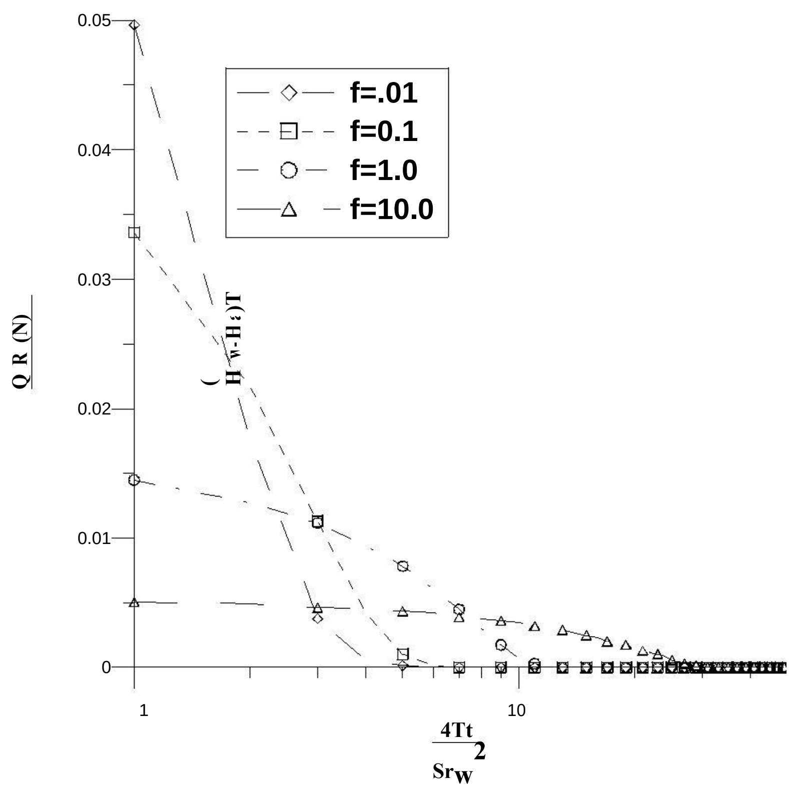

Figure 6. Non-dimensional recharge rate for different friction loss, k (per unit diameter).

Table 1. Hydro-geological data of the test problem.

Thickness of the aquifer	44.21 m
Initial Piezometric head above recharge well bottom level	110.75 m
Initial slug height	115.93 m
Diameter of the recharge well	0.35 m
Transmissivity	0.75 m^2/min
Storativity	0.000061
Friction parameter	0.01 to 10.0 m^{-1}

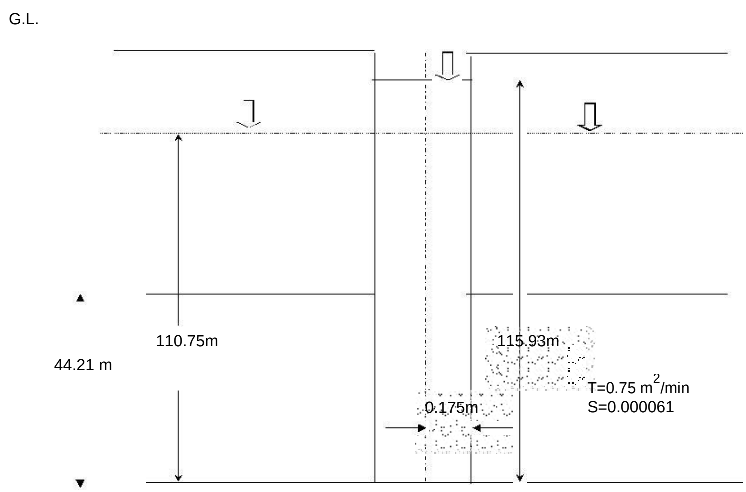

Figure 7. Conceptual hydro-geological aquifer configuration.

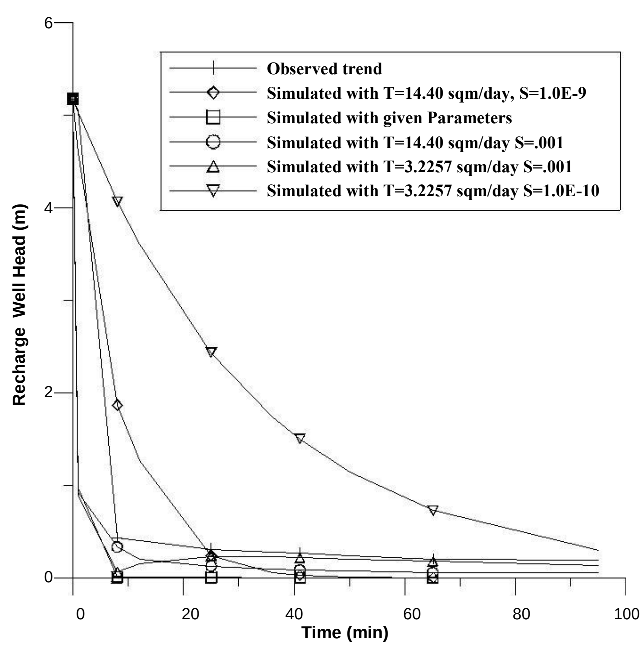

Figure 8. Recharge well water level with various T and S for Hansol case study.

Dissipation phase observations of 5.18 m well water column in the injection well are considered for the analysis. Initial water level in the well face is 12.56 m below ground level and water is poured in to the recharge well up to a level 7.38 m below ground level. Comparison of the estimated and the observed water levels are as shown in Figure 9 for different sets of parameters, T and S, including the set of parameters indicated by pumping tests. It is evident that no combination of T and S is able to produce an adequately good fit to the data for the entire time domain. Subsequently, in Figure 10, the fit is tried with the well loss parameter, C_w, as formulated in Appendix A, while in Figure 10, the fit is tried with the friction parameter, k. It is seen that the choice of friction parameter, k as the third parameter gives a better fit to the field data for the entire time period.

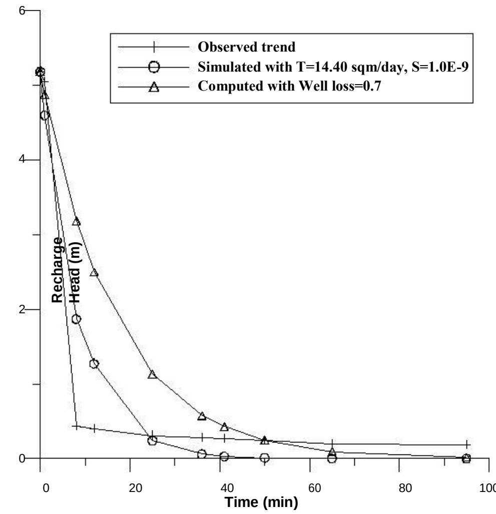

Figure 9. Recharge well water level trends with Walton well loss (min^2/m^5) for Hansol case study.

CONCLUSIONS

Analytical equations are developed for computing unsteady recharge rates and recharge well water levels during free recharge to a confined aquifer through a fully penetrating well. In the present equation, well storage is a function of aquifer diffusivity and recharge well water column, and well loss is time variant head loss due to inertial and frictional forces. When compared with the Walton head loss, field application indicates that friction parameter 'k' could be a third parameter other than T and S to simulate observed recharge well responses. Using the present equation, aquifer diffusivity can be found using only single information, that is, time taken to dissipate the initial water column in a recharge well of known radius. Any other well function can replace Theis well function so as to extend the technique to different hydro-geological conditions. Flexibility in transforming head in to flux and vice-versa at the well face makes it possible to generate pressure and recharge equations simultaneously.

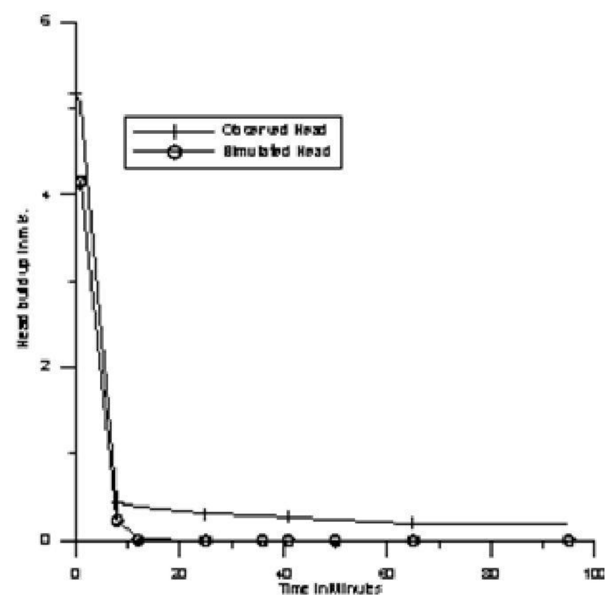

Figure 10. Recharge well water level trend with friction parameter (k) for Hansol case study.

Notations

H: Initial recharge well water column height (L), H_a: Height of piezometric level above datum (L), H_W: Height of well water level above datum (L), h: Hydraulic head (L), r_W: Well radius (L), r: Radial coordinate (L), D: Well bore diameter (L), s: Head buildup (L), h_f: Head loss due to friction (L), S_{Wl}: Well loss (L), S: Storativity (ratio), T: Transmissivity (L^2/T), Q_a: Rate of recharge at well face (L^3/T), Q_W: Change in well storage (L^3/T), V: Darcy flux (L^3/T), δ: Discrete Kernel ($L/(L^3/T)$), C_W: Well loss constant (T^2/L^5), f: Friction factor (non-dimensional), g: Acceleration due to gravity (L/T/T), R_e: Reynold's number (non-dimensional), ν: Coefficient of viscosity (L^2/T), k: Dimensional friction coefficient (/L), t: Time (T), Δt: Time step size (T), n: Time step count (number), γ: Time step count (number), τ: Time (T).

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APPENDIX A

Walton (1962) well loss in free RECHARGE

Well loss may be represented approximately by Jacob (1947) and Rorabaugh (1953):

$$S_{WL} = C_W Q^2$$

where S_{WL} is the well loss in m, C_W is the well loss constant in min^2/m^5 and Q is the discharge in m^3/min. Walton (1962) describes four conditions, which a well undergoes with its age. Well loss constants are mentioned to indicate well conditions. Considering mirror image of well loss for recharge well, head rise in the aquifer at the well face is written as:

$$s(r, n) = \sum_{\gamma=1}^{n} Q_a(\gamma)\delta(r, t, n-\gamma+1) - C_W (Q_a(n))^2 \quad \text{(a1)}$$

where C_W is the well loss constant.

Unlike pumping well, head loss in a recharge well may not be at a constant rate throughout the recharge cycle. Considering well loss to vary between two well conditions of Walton (1962), piezometric head rise in the aquifer at r = r_W is given by,

$$H_a(n) = H_a(o) + \sum_{\gamma=1}^{n} Q_a(\gamma)\,\delta(r, t, n-\gamma+1) - C_W(n)(Q_a(n))^2 \quad ; \; C_W > 0 \quad \text{(a2)}$$

Receding water level in the well may be expressed as:

$$H_w(n) = H_w(o) - \frac{1}{\pi r_w^2} \sum_{\gamma=1}^{n} Q_a(\gamma) \quad \text{(a3)}$$

At r = r_W, for each time step,

$$H_a(n) + C_W(n)\,(Q_a(n))^2 = H_w(n) \quad \text{(a4)}$$

Resulting quadratic Equation a5 can be solved for $Q_a(n)$.

$$[C_W(n)]Q_a(n)^2 + \frac{1}{\pi r_w^2} + \delta(r, t, 1)\,Q_a(n)$$
$$-\Pi_w(o) - H_a(o) - \frac{1}{\pi r_w^2}\sum_{\gamma=1}^{n-1} Q_a(\gamma) - \sum_{\gamma=1}^{n-1} Q_a(\gamma)\,\delta(r, t, n-\gamma+1) = 0 \quad \text{(a5)}$$