Global Science Research Journals

Available online at
www.globalscienceresearchjournals.org

Global Research Journal of Education

Full Length Research Paper Open Access

ISSN: 2408-6894, Vol. 8 (4). Pp. 1016-1027 June, 2020

The reasoning strategies used by mathematics teachers to promote students' thinking for the essential grades in Jordan: A mixed-methods study

Sumailah A. Sabbagh and Amjad M. Al-Khayat

Department of Educational Sciences, Salt Faculty Al-Balqa' Applied University, Salt, Jordan
Author Email: sumaila.sabbagh@gmail.com, am.alkhayat@bau.edu.jo

Accepted 1 November, 2019, Published June 24, 2020

Abstract

Reasoning in mathematics is the ability to understand mathematical concepts logically in order to form a judgment. The purpose of this study was to identify the reasoning strategies used by mathematics teachers at the elementary and middle stages in Jordan. To this aim, a mixed-methods study was conducted. A group of 12 teachers were observed during their teaching experience. Every class observation was observed against the common teachers' Reasoning Strategies Level Indicators. Qualitative data were analyzed using a content-analytical approach. The results revealed that the teachers do not commonly use reasoning strategies to promote students' thinking for both elementary and secondary schools in Jordan. Furthermore, the results revealed no significant difference in the common strategies used by the teachers in relation to grades, teachers' gender, and strategies levels. This study recommends organizing tailored training for mathematics teachers to equip them with the skills needed to implement reasoning strategies in-class and improve students' thinking. The study also urges mathematics teachers to be more aware of their role from being a carrier of information to being a supervisor and facilitator of the process of teaching towards more students' engagement in reasoning and thinking. Finally, replicating the current study with different stages is recommended.

Keywords: Reasoning Indicators, Essential grades; Mathematics Teachers; Jordan.

INTRODUCTION

Mathematics educators struggle to develop materials thatfoster powerful mathematical ideas, including learning to reason statistically, to think algebraically, to visualize, to solve problems, and to pose problems (English,2002). The current shifts in curricular and standards documents have set up new goals for the 21st century; warranting mathematics education to emphasize conceptual understanding, strategic competence, adaptive reasoning and justification, and productive communication (National Council of Teachers of Mathematics, NCTM, 2000:56; Ministry of Education in Jordan, MOD, 2013). The framework for mathematics curriculum development and implementation persists in including reasoning as a key component of mathematics education (NCTM, 2000:57). In fact, logical thinking and reasoning are deeply embedded in the culture of mathematics (Burton, 2004).

Part of becoming proficient in mathematics involve developing ideas, exploring phenomena, justifying results and using mathematical conjectures in all content area. Reasoning cannot simply be taught in a single unit on logic but should be a consistent part of students' mathematics experience in prekindergarten through grade 12. Reasoning mathematically is a habit of mind, and like all habits, it must be developed through using in many contexts (NCTM, 2000:56). According to the current document (NCTM), students at all levels should be able to communicate their mathematical thinking, analyze the thinking of others, use mathematical language to express ideas precisely, and develop and evaluate mathematical arguments and proof (NCTM, 2014).

There is a consensus that mathematics is more than procedural fluency; it is about knowing the “why” as well as the “how” of making connections with other mathematical ideas and everyday life. It is a prerequisite to any kind of skill development, specifically the fluency proficiency. Without understanding, lifelong mathematics learning will not be available for students (Lowe, 2013).

According to Sullivan (2012), there are four proficiencies (understanding, fluency, problem solving and reasoning), which together provide a clearer framework for mathematical processes than simply “working mathematically” and are more likely to encourage teachers and others who assess student learning to move beyond a focus on fluency, however, there will need to be support for teachers if they are to incorporate them into the curriculum (Sullivan,2012). Barmby et al. (2009) state “by developing the reasoning, we also develop the understanding. Drawing out children’s reasoning and developing the reasoning they use is therefore integral to developing understanding in mathematics” (Barmby et al.2009:6-7).

Lowe (2013), however, linked fluency to understanding, stating that fluency is about the capacity and ability of the learner to recall previously learned information readily “so that the skills that flow from understanding become habitual and the learner can use them to proceed to higher levels” (p. 11). Atweh et al. (2012, p. 13) argued a “shift in focus from what knowledge and skill is required in/by mathematics and in schools to a focus on what is required for a citizen to become a confident and effective user of mathematics in society” was necessary. The proficiencies encouraged this, particularly the reasoning and problem solving proficiencies.

The Reasoning Proficiency is about children making sense of the mathematics by explaining their thinking, giving reasons for their decisions and describing mathematical situations andconcepts. Children need to be able to speak, read and write the language of mathematics. Important mathematical reasoning language includes the language of thinking, the language of justification and the language of proof.

In contrast, Askew (2012) identified “understanding” as(a): building robust knowledge of mathematical concepts, and (b) making connections between related ideas. He also could not see the difference from the explaining or justifying in the reasoning proficiency, or with communicating solutions in problem solving. He concluded that “a good balance of the actions involved in fluency, problem solving, and reasoning will lead to understanding” (Askew, 2013:20-29).

Reform-oriented teachers are focused on students gaining deeper understandings of mathematical ideas, relations, and concepts rather than focusing just on accuracy (Kazemi&Stipek, 2001). According to Stein, mathematics should be taught in a way that encourages students to make conjectures, talk, question, and reason in orderto get deep understanding (Stein,2007). Reasoning is a necessary condition in an analysis of knowledge (Greenbush & Pritchard, 2009). While Knuth and others stated that mathematical proficiency can be described as five strands of students’ cognitive engagement: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Knuth, et al., 2009).

Jordan has been a member of the Programmers for International Student Assessment (TIMSS) and The Programmers for International Student Assessment (PISA) for two years. The mathematics proficiency of students in Jordan, as measured by (TIMSS) lags behind the performance of students worldwide (TIMSS, 2015). This result is in line with PISA results which ranks Jordan 51 out of 70 and has remained stagnant for many years (PISA, 2016).

Mathematical Reasoning

Mathematical reasoning can be conceptualized as the ability to understand and make sense of mathematical concepts logically in order to form a conclusion or judgment (Merriam-Webster, 2014). It also includes “being able to reason is essential to understanding mathematics” (NCTM, 2000:3).

Mathematical reasoning involves comprehending mathematical information and concepts logically forming conclusions and generalizations based on this comprehension. It provides students with the ability to comprehend all other aspects of mathematics as individuals recognize that mathematical skills and concepts to make sense by exploring patterns or regularities, synthesizing information, and providing arguments to support their conclusions (NCTM, 2000:55).

The Standards for Mathematical Practice outlined in the Core Curriculum State Standards for Mathematics (CCSSM) emphasize the conceptual understanding of mathematics and mathematical along with procedural fluency and problem-solving, as well as encourage the use of technology and tools within the classroom to further mathematical understanding (CCSSM, 2010).

Researcher argue from a different perspective, highlighting three levels of reasoning that merit special attention for school mathematics; the power of reasoning and justification to establish certainty, its ability to communicate mathematical thinking, and the authenticity of mathematical experience that it can provide (Thompson et al., 2012). Reasoning plays a central role in mathematical problem solving, and to that end has called mathematics teachers and mathematics education researcher to focus their attention on helping students become better thinkers.

Reasoning requires command of mathematical skills but researcher reflect that even students with strong math skills may come up short in reasoning from inability to translate that strength into an argument. In addition, it involves application in context (Bok, 2006), and communication (Brakke, 2003). It may be easy to see that effective communication in the twenty-first century requires facility with reasoning (Wolfe,2010). Finally, reasoning describes as “a habit of mind rather than a set of topics or a list of skills” (Hughes Hallett, 2003: 91).

Teacher's Role in improving students' learning

Teachers are considered at the forefront to (a) generate and support students' engagement with mathematics; (b) foster student deep conceptual understanding; ; (c) help students develop the ability to formulate problems, and explore ,conjecture, and reason logically, and (d) establish a classroom environment in which students' value and engage in mathematical reasoning (Bruce, 2007). This comes in line with the results of many recent studies in Jordan aiming to find the causes ofdifficulties in learning mathematics. These studies showed that teachers think their responsibility is to transmit their knowledge and mathematical laws to the students over-rote memorization, a teaching strategy that made many students suffer from understanding basic mathematical concepts, weakness in their ability to solve math problems and low level of communication skills (Balas & Barhem,2010; Sabbagh,2007; Alali,2001).

Norton claims that students as early as the primary grades, can and do engage in making and refuting claims, use both inductive and deductive modes of reasoning, and generally treat mathematics as sense-making activity (Norton, 2010). Francisco and Maher (2010) have also found that teachers often underestimate students' mathematical reasoning abilities, and subsequently these abilities are underutilized as a pathway to mathematical thinking by both teachers and students.

Yakel stresses that teachers need to support students’ reasoning as they interact around developing arguments, and eliciting or providing missing warrants and backings for claims, and to understand both the underlying mathematical concepts and conceptual terrain that is open to their students (Yakel, 2002). What a teacher consider as normative in the classroom context, both for his own role and for what they expectof his students, can affect the opportunities for reasoning that perceive and utilize the kinds of norms he attempts to negotiate with his students to develop reasoning (Harel & Rabin, 2010). In addition, the teacher needs to appropriately guide discussion, and be able to evaluate an argument’s validity. Researcher have found that when teachers pose open-ended tasks, make students responsible for reasoning and guide them as they do so, and analyze the other students produce, the resulting classroom environment is conducive to productive student engagements with reasoning (Lloyd, 2005; Carrier, 2014).

Therefore, the current study aims to examine thereasoning strategies employed by middle and elementary teachers in Jordan to promote students’ thinking and reasoning strategies and explore whether those strategies meet the requirements of the 21st Century standards for mathematics teaching.

REVIEW OF LITERATURE

Carrier (2014) conducted a study to recognize indicators of multiplicative among fourth-grade students. Through cross-case analysis, the researcher used a test instrument to observe patterns of multiplicative at varying levels in a sample of 14 math students from a low socioeconomic school. Results indicate that the participants fell into three categories: pre-multiplicative, emergent, and multiplier. Consequently, 12 new sublevels were developed that further describe the multiplicative thinking of these fourth graders within the categories mentioned. Rather than being provided the standard mathematical algorithms, students should be encouraged to personally develop their own unique explanations, formulas, and understanding of general number system mechanics. When instructors are aware of their students’ distinctive methods of determining multiplicative strategies, they are more apt to provide the most appropriate learning environment for their students (Carrier, 2014).

Alzoubi’s (2014) study aimed at investigating the effect of a teaching strategy based on problem solving on developing the mathematical creative thinking skills for the class teacher students. The sample of the study were (98) students, distributed randomly into control group taught by the traditional strategy, and the experimental group taught by a strategy based on problem solving. A mathematical creative thinking test was constructed by the researcher, and used as a pre-post test. Results showed a significant difference between the means of creative thinking for the experimental and control group, in favor of the experimental group (Alzoubi,2014). The study for AbouEid and Jaradat (2016) aimed at revealing the effect of using learning teaching strategy based on social interaction through cooperative learning in developing verbal communication skills of sixth grade students in Jordan. To achieve this objective, Learning teaching strategy was developed and applied on a sample of 112 students were divided into two groups: an experimental group taught by suggested strategy; and controlled group taught without suggested strategy. A verbal communication skills test in mathematics was applied as a pre and post test. Also a 2-way ANOVA were applied. The study results revealed that there were statistically significant differences between the two groups in favor of the experimental group (AbouEid & Jaradat, 2016).

Khamees's (2012) study aimed to reveal the impact of the proposed training program for the development of mathematical thinking for seven grade math students. To achieve this purpose, a study sample of (182) students from Seventh grade students divided into two groups: experimental group that taught using the training program, and the control group that taught using traditional method. An achievement test in mathematics, were used after the implementation of the study directly, and after four Weeks of implementation. To answer the questions of the study two-way analysis were used. The results showed the significant positive impact of the program in developing mathematical thinking at the level (0.05= α) in the direct and delayed achievement in mathematics for both male and female students (Khamees, 2012). Gawain's (2011) study attempted to measure context-rich application of quantitative. Initial evidence suggests that teaching to explicitly state learning goals—whether in the context of a limited cluster of courses or in offerings across the curriculum—can shape student aptitude (Grwain, 2011). A study for Thompson and colleagues (2012) aimed to find out the opportunities to learn reasoning and proof in High School Mathematics Textbooks. This study addresses the nature and extent of reasoning and proof in the written curriculum of 20 contemporary high school mathematics textbooks. Both the narrative and exercise sets in lessons dealing with the topics of exponents, logarithms, and polynomials were examined. The result appeals, about 50% of the identified properties in the 3 topic areas were justified, with about 30% of the addressed properties justified with a general argument, and about 20% justified with an argument about a specific case. However, less than 6% of the exercises in the homework set involved proof-related, with developing an argument and investigating a conjecture as the most frequently occurring types of proof-related (Thompson et al, 2012).

Grant and colleagues (2007) discussed the challenges and opportunities that arose in attempting to support prospective elementary teachers in developing mathematical justifications in the context of whole number computation. The paper discusses the importance of justification, and clarify the question to whom must the argument be convincing? The first reasoning may convince those who already have knowledge of the distributive property but would likely be meaningless to elementary school students. The second reasoning would likely be more convincing to elementary school students because it relies on a typical interpretation of multiplication. Because our main goal is to prepare future teachers, we seek to deepen their understanding of operations by promoting the kind of illustrated in the second reasoning (Grant, et al, 2007). Morris (2007) has documented how pre-service elementary teachers' criteria for evaluation student arguments in a classroom transcript can shift from favoring example-based inductive arguments to favoring arguments based on a key idea, with the shift associated with the inclusion of a valid deductive argument in the transcript (Morris, 2007). While Knuth found that teachers' limited conceptions of reasoning relegated it to the role of just another topic to be learned. This may tend to turn reasoning tasks into exercises in writing statements in a strictly prescribed form rather than acts of explanation and communication (Knuth, et al,2009). Morris has described the use of consistent and powerful representation in a Russian elementary –level curriculum. These representations provide students with a platform for about generalized relationships among quantities and thus facilitate their engagement with reasoning (Morris, 2009). Similarly, Schifter emphasized the power of representation to support reasoning for elementary level students (Schifter, 2009).

The study of Sansome (2016)explored the ways in which teacher practices, when focused on reasoning, enhance the disposition of students towards greater mathematical proficiency. This research addressed significant problems and worked with teachers rather than studying them to developed new ways of seeing/theorizing mathematics teaching and learning that has left a foundation for reasoning to impact teaching and learning. Major findings indicate that specific practices, such as questioning, journaling and discussion, work to benefit students' reasoning abilities and dispositions in primary mathematics classrooms. Lloyd (2005) pointed out that teacher preparation and professional development programs must offer teachers the resources and guidance they need to successfully engage their classes with justification, and this will help the teachers in students-centered mathematics classroom so that teachers have ideas of what to do and not simply admonitions against what teachers should not do (Lloyd, 2005). Lithner (2008) has created a research framework for different types of mathematical reasoning, distinguishing between two main types: imitative and creative. Imitative is rote learnt, while creative is based on mathematical foundations. One of the main differences between imitative and creative is that the former does not necessaril1y involve analytical and conceptual thinking, whereas such thinking processes are essential to creative (Lithner, 2008).

Although each of these different ways of examining reasoning offers important information about the experiences of mathematics to students and teachers, there is still a looming concern regarding teacher's role in using different levels of reasoning for their students. Indeed, there is some evidence that even with using reform curricula which included a standard across all grade bands, students continue to struggle with mathematical reasoning (Thompson, et al,2012; TIMSS ,2015; Kazemi & Stipek, 2001; Stein,2007; Greenough & Pritchard,2009).

Rationale of the Study

Though curriculum development has a potential to enhance students' engagements with justification, it is still fundamentally important to consider the role of teachers. This study investigates reasoning strategies used by elementary and middle school teachers in Jordan. To the best of our knowledge, no studies have yet systematically investigated these strategies of reasoning used by the teachers across the upper elementary (grades 4–6), and middle (grades 7–9) stages in Jordan. The majority of studies were experimental studies to improve mathematical thinking for the students. Although there have been few research studies on teacher's role in applying reasoning strategies, there is still a need to investigate the strategies across a range of ages and to explore the corresponding stages of developmental trends. The body of research on difficulties that students and teachers encounter with reasoning is extensive. This study will investigate the common reasoning strategies that teachers used to develop students' thinking across a larger range of grade levels (grades 4–9).

Study Questions

The overarching goal of the study was to investigate and achieve an in depth understanding of the reasoning strategies used by elementary and middle stage teachers to promote students' thinking in Jordan. In specific, the study attempted to answer the following research questions:

1. What are the most common reasoning strategies employed by teachers of the elementary stage in Jordan?
2. What are the most common reasoning strategies employed by middle grades teachers of the middle stage in Jordan?
3. Is there a significant difference at (α=.05) between reasoning strategies employed by middle grades teachers in Jordan depending on the classroom level and teacher's gender?
4. Is there a significant difference at (α=.05) between the means of reasoning strategies employed by elementary grades teachers and middle grade stages in Jordan depending on the reasoning levels?

METHODS

The study used a data triangulation approach to collect both quantitative and qualitative data.

Qualitative data are highly descriptive, and in order to interpret the information, the data need to be reduced. In this study, a content-analytical approach was chosen for this purpose. The basic idea of content analysis is to take texts and analyze, reduce and summarize using emergent themes. These themes can then be quantified, and as such, content analysis is suitable for transforming textual material into a form, which can be statistically analyzed (Cohen, 2007).

The Study Instrument (RLS)

The study used the common teachers' Reasoning strategies Level Indicators (RLS) The RLS is a formative assessment focusing on core strategies to assess mathematical reasoning. It was used as a resource for considering the conceptualization of mathematical reasoning. The theory behind that was related to two main dimensions, including content and cognitive engagement. Cognitive engagement is considered as essential dimension for mathematical proficiency as noted in the work of Kilpatrick et al. (2001) and is discussed in detail in the technical manual (Bernbaum, Wilmont, 2012).

Ball and Bass (2003) highlighted three levels of reasoning that merit special attention for school mathematics; the power of and Reasoning to establish certainty, its ability to communicate mathematical thinking, and the authenticity of mathematical experience that it can provide (Ball & Bass, 2003). Depending on that, the researcher constructs the (RLS) tool using these three levels indicators as follows: the power of reasoning establishes certainty (PR skills). The ability to communicate mathematical thinking (COM skills) and finally the authenticity of mathematical experience that it can provide different skills to construct reasoning and proof in different ways. (PC Skills).

The instrument used five-point likert scale to rate teachers' observation as follows: 1 (strongly disagree), 2(disagree), 3(uncertain), 4 (agree), 5(strongly agree). The instrument (RLS) has been used by the researcher and the co researcher team to analyze thedata collected from observations .

Procedures for the Study

To answer the question of the study, the researchers followed these steps : The researchers purposively selected 12 sites, including government and UNRWA schools in Amman . A total of 12 basic school teachers were included in the study. A team of 12 research assistants,who were enrolled in math teaching course on Math Department in the University of Jordan, conducted the data collection. All research assistants were trained by the study PI in order to ensure a unified methods of data collection, recording, and analysis. Documents of written observation were analyzed by the researchers and research assistants using the preliminary categories as the coding scheme. Then each member of the co researchers selected randomly (2) teachersto observe during math classes. After each observation, the observer coded,analyzed and reported the data using (RLS). Two observation documents were analyzed by one researcher and one of the research assistants tocalculate the ratio of agreement. The average of theagreement ratio was 92%, which was deemed acceptable for the purpose of this study. Each teacher was observed frequently as shown in Table 1.

The second round analysis showed that the categories found in the initial phase were sufficient for covering all reasoning levels found in the tool. A quantitative approach was then taken in order to be able to illustrate the results. The use of both quantitative and qualitative methods has several benefits. Mixed methods avoid any potential bias originating from using one single method, as each method has its strengths and weaknesses. A mixed methods approach also allows the researcher to analyze and describe the same phenomenon from different perspectives and exploring diverse research questions. Whereas questions looking to describe a phenomenon (``How/What..?") are best answered using a qualitative approach, quantitative methods are better at addressing more factual questions (``Do...") (Cohen, 2007).

Table 1: Number of Observers and Observations Per Grade

Grades	Fourth	Fifth	Sixth	Seventh	Eight	Ninth
No. of observers	5/ male	5/ male	5/ male	6/ male	5/ male	5/ male
	6/Female	5/Female	5/Female	6/Female	5/Female	5/Female
Sum of Observation	11	10	10	12	10	10

Data Resources

Data were collected contextually, in the natural setting in the light of the researchers' observation, which focused on highlighting the reasoning strategies to improve students' understanding and thinking during the teaching procedures of the participants. The process of collecting data continued for three months during the first semester of the academic year (2017-2018) .

Subjectivity and Objectivity of the Collecting Data

To guarantee objectivity, the researcher applied the following methods: objective narration of events,long-time field observation and the triangular data collection method. Collection of data relied on field observation at different locations and times. Such methods of collection, describingand analyzing the data have been commended by many researches in the field. The collection and processingof the data passed through the following four stages: planning, developing intimate relations with the participant, collection of raw data, and finally summarizing, coding, and analyzing the collected data .

Data Analysis

Researcher used inductive analysis to arrive at the results of the data collected. The study adopted thedescriptive design using percentage and averages for each level of the reasoning from all sites .

After converting the qualitative data into quantitative data through analyzing and recording the results using the study tool (RLS). The means and standard deviations for each level of reasoning have been calculated. Also, two way and one way ANOVA have been calculated to determine whether the differences between means of different classes, gender, and levels of reasoning are statistically significant at the level (α=.05).

RESULTS

Four research questions guided this study. Answers were obtained through inductive analysis for the data collected from the observation in the natural setting. The researchers observed the participant in the classroom and monitored the strategies used by the teachers.

Question 1: What are the most common reasoning strategies employed by teachers of the elementary stage in Jordan?

The data collected from the fourth, fifth and sixth grades revealed that the three most common reasoning strategies were asking students to clearly identify the hypotheses (40%); developing a mathematical argument depending on a representation (37.1%); and giving the students the opportunity to guess consciously before starting the solution (35.8%). These were at the first and second reasoning level. On the other hand, the least three used reasoning strategies were selecting tasks that develop reasoning (27.9%); using different reasoning strategies as a model for students (27.9%); and generalizing arguments for a broader category of problems and making links between these problems (27.9%). These were from the first and third levels of reasoning. Overall, reasoning strategies were not commonly used (less than 40%). Details regarding all used strategies are presented in Table 2.

Table 2: Reasoning strategies employed by Mathematics Elementary Teachers in Jordan

Percent %	Total Sum	Reasoning Strategy	Reasoning Level	Strategy No.
27.9	88	Selects the tasks that develop reasoning	1st	7
27.9	88	Uses different reasoning strategies as a model for his students	1st	8
27.9	88	Generalizing arguments for a broader category of problems and making links between these problems	1st	20
28.2	89	Constantly asking students to justify their answers	4th	2
28.5	90	Asses inductively.	5th	12
28.5	90	Evaluation a deductive arguments or part of them	5th	17
30.7	97	Constantly asking how you got the answer	1st	1
31.4	99	Development of deductive arguments or part of them	3rd	16
31.5	100	Expands the discussion between students during explanation and solution	1st	5
32	101	Explanation of the solution and how the answer reached a decision under unspecified conditions	1st	9
32.3	102	Developing inductive arguments.	2nd	11
32.6	103	Student asked to make conjectures	1st	4
33.6	106	The classroom discussions give students the opportunity to justify and link up what they learn	2nd	15
33.9	107	Consider the reasonableness of the answer and determine the logical level of accuracy	1st	10
33.9	107	Justifying fallacies in the solution or proof.	2nd	13
34.9	110	Uses a wide range of assessment tools that develop the student's ability to justify such as asking students to explain, give example, non example, representation,	3rd	18
35.2	111	Development of proof through the counterexample	3rd	19
35.8	113	Give the students the opportunity to guess consciously before starting the solution	1st	6
37.1	117	Develop a mathematical argument depending on a representation.	2nd	14
40	126	The student is asked to clearly identifying the hypotheses.	1st	3

Question 2: What are the most common reasoning strategies employed by middle grades teachers of the middle stage in Jordan?

The data collected from the seventh, eighth and ninth grades revealed that the three most common reasoning strategies were expanding the discussion between students during explanation and solution (40.6%); constantly asking how did you get the answer (40%); and having the classroom discussions giving students the opportunity to justify and link up what they learn (39%). These were at the first and second reasoning level. On the other hand, the least three used reasoning strategies were assessing inductively (31.1%); development of proof through the counterexample (32%); and developing inductive arguments (32.6%). These were from the second and third levels of reasoning. Overall, reasoning strategies were not commonly used (less than 41%). Details regarding all used strategies are presented in Table 3.

Table 3: Reasoning strategies employed by Mathematics Middle Stages Teachers in Jordan

Total Sum	Total Sum	Reasoning Strategy	Justification Level	Strategy No.
31.1	98	Asses inductively.	2nd	12
32	101	Development of proof through the counterexample	3rd	19
32.6	103	Developing inductive arguments.	2nd	11
33	104	Uses different Reasoning strategies as a model for his students	1st	8
33	104	Justifying fallacies in the solution or proof.	2nd	13
33.6	106	Development of deductive arguments or part of them	3rd	16
34.2	108	The student is asked to clearly identifying the hypotheses..	1st	3
34.2	108	Evaluation a deductive arguments or part of them	3rd	17
34.6	109	Student asked to make conjectures .	1st	4
34.9	110	Explanation of the solution and how the answer reached a decision under unspecified conditions	1st	9
34.9	110	Generalizing arguments for a broader category of problems and making links between these problems	3rd	20
35.2	111	Selects the tasks that develop understanding and justification	1st	7
35.2	111	Consider the reasonableness of the answer and determine the logical level of accuracy	1st	10
35.5	112	Develop a mathematical argument depending on a representation.		14
37.4	118	Constantly asking students to justify their answers	1st	2
37.4	118	Uses a wide range of assessment tools that develop the student's ability to justify such as: asking students to explain, give example, non example, representation,	3rd	18
37.7	119	Give the students the opportunity to guess consciously before starting the solution	1st	6
39	123	Expands the discussion between students during explanation and solution	1st	5
40	126	Constantly asking how did you get the answer	1st	1
40.6	128	The classroom discussions give students the opportunity to justify and link up what they learn	2nd	15

Question 3: Is there statistically significant differences between reasoning strategies employed by teachers depending on the grades and gender at (α=.05)?

To investigate the difference, 2-way ANOVA was conducted. The analysis included the main effects of gender and grades as well as their interaction. The dependent variable included the percentage of employed reasoning strategies. Table 4 shows the results, which revealed no statistically significant differences between justification strategies employed by teachers depending on their gender and grades at (α=.05). In other words, the reasoning strategies employed by teachers is the same no matter what is their gender or the grades they teach.

Table 4: Two-way ANOVA exploring the effect of grades and gender on teachers' reasoning strategies

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Grades	675.966	5	135.193	.722	.610
Gender	648.928	1	648.928	3.465	.068
Grades * Gender	452.554	5	90.511	.483	.787
Error	9550.567	51	187.266		
Corrected Total	11377.079	62			

Question 4: Is there a significant difference at (α=.05) between the means of reasoning strategies employed by elementary grades teachers and middle grade stages in Jordan depending on the reasoning levels?

To investigate the difference, One-way ANOVA was used. The results revealed no significant differences. Table 5 presents the results. There was no statistically significant difference between reasoning strategies employed by elementary grades teachers and middle grades. The first level which is concerned about realization of the importance of mathematical reasoning and proof was not significantly different than the second level that illustrated the presentation of inductive arguments (relying on special cases) or the third level which deal with the presentation of deductive arguments. This result reflects that teachers' randomly select some reasoning strategies with no defined aim of improving students' thinking and reasoning.

Table 5: One-way ANOVA exploring the difference in reasoning strategies levels used by elementary and middle grades teacher

Justifications' Level		Sum of Squares	df	Mean Square	F	Sig.
Level 1	Between Groups	311.802	5	62.360	1.270	.290
	Within Groups	2799.912	57	49.121		
	Total	3111.714	62			
Level 2	Between Groups	72.184	5	14.437	.695	.629
	Within Groups	1183.562	57	20.764		
	Total	1255.746	62			
Level 3	Between Groups	43.711	5	8.742	.602	.699
	Within Groups	828.003	57	14.526		.290
	Total	871.714	62		1.270	

DISCUSSION

This is the first study to investigate and achieve an in depth understanding of the reasoning strategies used by elementary and middle stage teachers to promote students' thinking in Jordan.The research findings that emerged from the observations showed that teachers do not commonly use reasoning strategies to promote students' thinking for both elementary and secondary schools in Jordan. In fact, the use of these strategies, even when present, did not seem effective. For example, the item “Uses different reasoning strategies as a model for students” scored a relatively low percentage (33%), indicating that teachers are not aware of the different methods needed to promote students’ thinking. These results are not consistent with many previous research studies and theoretical literature that concludes reasoning as a necessary condition in the analysis of knowledge.

We highly recommend that the Jordanian Ministry of Education train teachers to utilize the four proficiencies (understanding, fluency, problem solving and reasoning), which provide a clearer framework for mathematical processes than simply “working mathematically” and are more likely to encourage teachers and others who assess studentlearning to move beyond a focus on fluency. Such strategies should be incorporated into the teaching curricula and their effectiveness should be evaluated in longitudinal studies. When teachers are aware of their students’ distinctive methods of determining multiplicative strategies, they are more apt to provide the most appropriate learning environment for their students (Carrier, 2014).

Furthermore, the results revealed no significant difference in the common strategies used by the teachers in relation to grades, teachers' gender and strategies levels. This also reflects what manyrelated research conclude that most mathematics teachers no matter what their gender or teaching grades, they perceive mathematics as a rigid and fixed body of knowledge, and they think that their responsibility is to transfer information and mathematical laws to the students over rote memorization, and concentrate only on theprocedures. Such traditional approach is often correlated with students suffer in their attempt to understand basic mathematical concepts, and causes weaknesses intheir ability to solve math problems and low level of reasoning skills (TIMSS, 2015; PISA, 2015; Norton, 2010; Lowe, 2013; Askew, 2012; Balas & Barhem; 2010, Sabbagh, 2007; Alali, 2001).

Not only this study revealed that teachers underuse reasoning strategies in their teaching, but also they underestimate students' mathematical reasoning abilities, and subsequently these abilities are underutilized as a pathway to mathematical thinking. Many researchers believe that teacherstend to use a transmission style of classroom communication, that is they resort to stating information rather than developing reasoning, and offering little opportunity for students to justify, explore, or make meaning forthemselves in order to apply reasoning and thinking (Francisco & Maher, 2010; Yakel,2002).

RECOMMENDATIONS

This study recommends implementing mathematical teaching methods that are compatible with mathematical reasoning and thinking. In addition, The Ministry of Education in Jordan should organize tailored training for mathematics teachers to equip them with the skills needed to implement reasoning strategies in-class and improve students’ thinking. The study also urges mathematics teachers to be more aware of their role from being a carrier of information to being a supervisor and facilitator of the process of teaching towards more students’ engagement in reasoning and thinking. Finally, replicating the current study with different stages is recommended.

Acknowledgment: Special thanks are to the mathematics students at Al-Balqa University who helped in data collection and entry.

Conflict of interest: None

Funding: None

REFERENCES

AbouEid, Ahmed &Jaradat, Maher(2016).The Effect of Using Learning Teaching Strategy Based on Social Interaction Through Cooperative Learning in Developing Verbal Communication Skills of Sixth Grade Students in Jordan. Psychological & Educational Studies Review, NO 16 , June 2016.

Al-Khateeb, Mohammad and Abadneh, Abdalla (2011). Thinking and Attitudes towards Mathematics with Seventh Graders in Jordan .Darasat for Educational Studies. Volume No. 38 (1): 189-204 .

Alali, Omar (2001).The acquisition of tenth grade students the skills of estimation, mathematical communication, patterns and mathematical relationship. Unpublished MA Thesis, armouk University, Irbid, Jordan.

Askew, M. (2012). Professor’s page: Is understanding a proficiency? Australian

Primary Mathematics Classroom, 17(1):20-30.Retrieved from http://www.aamt.du.au /Webshop/ Entire-catalogue/Australian-Primary-Mathematics-Classroom.

Alzoubi, Ali (2014). The Effect of a Teaching Strategy Based on Problem Solving on Developing the Mathematical Creative thinking Skills for the Class Teacher Students, Jordanian Educational Journal, No. 10:305-320.

Atweh, B., & Goos, M. (2011). The Australian mathematics curriculum: A move forward or back to the future? Australian Journal of Education, 55(3), 214–228.

Atweh, B., Miller, D., & Thornton, S. (2012). The Australian Curriculum: Mathematics –world class or déjà vu. In B. Atweh, M. Goos, R. Jorgenson, & D.

Australian Curriculum and Reporting Authority [ACARA]. (2014). The Australian Curriculum: Mathematics. Retrieved fromhttp://www.australian curriculum .edu.au/mathematics/
Australian Curriculum, Assessment and Reporting Authority. (2010). Australian Curriculum: Mathematics. Retrieved from ttp://www.australiancurriculum.edu.au/mathematics/content-structure
Balasm Ryad; Barhem, Areej(2010). The effect of using multiple mathematical representations for the eighth grade students to gain the Basic mathematical concepts and their ability to solve word problems, Dirasat, educational science, University of Jordan. Vol.37 (1): 1-13. .
Ball, D. L., & Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), A research companion to Principles and Standards for School Mathematics (pp. 27-44). Reston, VA: National Council of Teachers of Mathematics.
Barmby, P., Bilsborough, L., Harries, T., & Higgins, S. (2009). Primary mathematics: Teaching for understanding [EBL version]:6-7.Retrieved from http://qut.eblib.com.au/patron /Full Record .aspx?p=480596
Bernbaum Wilmont, D. (2012). Math inventory. (Technical Report) Sausalito, CA: Math Solutions, Scholastic.
Bok, D. Our Underachieving Colleges: A Candid Look at How Much Students Learn and Why They Should Be Learning More. Princeton, N.J.: Princeton University Press, 2006.
Brakke, D. F.(2003). “Addressing Societal and Workforce Needs.” In B. L. Madison and L. A. Steen (eds.), Quantitative Literacy: Why Numeracy Matters for Schools and Colleges. Princeton, N.J.: National Council on Education and the Disciplines.
Bruce, C. D. (2007). Student interaction in the math classroom: Stealing ideas or building understanding .Retrieved from http://www.edu.gov. on.ca/eng/literacy numeracy /inspire/research/Bruce.pdf.
Burton, L. (2004). Mathematicians as enquirers: Learning about learning mathematics., Boston: Kluwer Academic Publishers.
Carrier, Jim (2014) . Student Strategies Suggesting Emergence of Mental Structures Supporting Logical and Abstract Thinking: Multiplicative School Science and Mathematics. Volume 114 (2) :87-96
Cohen, L., Manion, L. & Morrison, K. (2007) Research Methods in Education. 6th Edition. London: Routledge New York.
Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics (CCSSM). .http://www.corestandards .org/wp-content/uploads/Math.
De Lange, J. “Mathematics for Literacy.” In B. L. Madison and L. A. Steen (eds.), Quantitative Literacy: Why Numeracy Matters for Schools and Colleges. Princeton, N.J.:
English, L.D.:(2002). ‘Priority themes and issues in international research in mathematics education’, in L.D .English, M.-G. Bartolini Bussi, G. Jones, R.A. Lesh and D. Tirosh (eds.), Handbook of International Research in Mathematics Education, Lawrence Erlbaum, Mahwah, NJ, pp. 3–15 .
Francisco, J. M., & Maher, C. A. (2010). Teachers attending to students’ mathematical : Lessons from an after-school research program. Journal of Mathematics Teacher Education, 14(1), 49-66.
Greenough, P., & Pritchard, D. (Eds.). (2009). Williamson on Knowledge. Oxford.
Grawe, Nathan, D (2011). Beyond Math Skills: Measuring Quantitative in Context. NEW DIRECTIONS FOR INSTITUTIONAL RESEARCH, no. 149:41-52 , Spring 2011 © Wiley Periodicals, Inc.
Harel, G., & Rabin, J. M. (2010). Teaching practices associated with the author-
itative proof scheme. *Journal for Research in Mathematics Education*, *41*:14-19
Hughes Hallett, D.(2003) “The Role of Mathematics Courses in the Development of Quantitative Literacy.” In B. L. Madison and L. A. Steen (eds.), Quantitative Literacy: Why Numeracy Matters for Schools and Colleges. Princeton, N.J.: National Council on Education and the Disciplines. p: 91-
Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper elementary mathematics classrooms. Elementary School Journal, 102(\), 59-80
Khamees,N.M (2012). The impact of a Training Program for the Development of Mathematical Thinking in the Collection Seventh Grade Students in Math. Damascus University Journal, No.28, volume:2.
Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). Adding it up: Helping children learn mathematics. Washington, DC: National Academy Press.
Knuth, E. J., Choppin, J. M., &Bieda, K. N. (2009). Middle school students' production of mathematical justifications. In D. A. Stylianou, M. L. Blanton,& E. J. Knuth (Eds.), Teaching and learning proof across the grades: A K-16 perspective (pp. 153-170). New York: Routledge.
Kramarski, B., & Zeichner, O. (2001). Using technology to enhance mathematical: Effects of feedback and self-regulation learning. Educational Media International, 38(2/3), 77-82.
Lithner, J. (2008). A Research Framework for Creative and Imitative . Educational Studies in Mathematics, 67, 255-276.
Lloyd, G. M. (2005). Beliefs about the teacher's role in the mathematics class-room: One student teacher's explorations in fiction and in practice. Journal of Mathematics Teacher Education, 8 , 441-467.
Lowe, I. (2013). AusVELS Mathematics. Vinculum 50(3), 10–13. Retrieved from http://www.mav.vic.edu.au/
Merriam Webster Online. Retrieved July 7, 2014, from http://www.merriam-webster.com/dictionary/reasoning
Ministry of education in Jordan, (2013). The General Framework for Mathematics Curriculum for primary andsecondary in Jordan, Amman, Jordan .
Morris, A. K. (2007). Factors affecting pre-service teachers' evaluation of the validity of students' mathematical argument in classroom contexts. Cognition and Instruction,25:479{522.
Morris, A. K. (2009). Representations that enable children to engage in deductive argument. In D. A. Stylianou, M. L. Blanton, & E. J. Knuth (Eds.), Teaching and learning proof across the grades: A K {16 perspective (pp. 87{101). New York: Routledge.
National Council of Teachers of Mathematics. (2000). Guiding principles for mathematics curriculum and assessment. Reston, VA: NCTM
NCTM (2008). Standards for School Mathematics. Available online: http://standards.nctm.org. Accessed on June 12, 2008.
National Council of Teachers of Mathematics. (2011). Principles and standards for school mathematics. Available online at https://www.usi.edu /science /math/ sallyk/Standards/document/chapter3/reas.htm
National Council of Teachers of Mathematics (NCTM). 2014. Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM.
Norton, S. (2010). How deeply and how well? How ready to teach mathematics after a one year program? Mathematics Teacher Education and Development, 12(1), 65–82.
Rutz, C. A., and Grawe, N. D.(2010). “Pairing WAC and Quantitative Reasoning Through Portfolio Assessment and Faculty Development.” Across the Disciplines, 2009, 6(Dec.). Retrieved June 11, 2010, from http://wac.colostate.edu/atd/assessment/rutz_grawe.cfm.
Sabbagh,S.A.(2007). Mathematical Communication Strategies Used by Talented Students in the Upper Basic StageinJordan.Dirasat, Educational Science, University of Jordan .Vol:34, No.7:302-318 .
Schifter, D. (2009). Representation-based proof in the elementary grades. In D.A. Stylianou, M.L. Blanton, & E.J. Knuth (Eds.),Teaching and learning proof across the grades: A K{16 perspective (pp. 71{86). New York: Rout-ledge.
Staples, M. (2007). Supporting whole-class collaborative inquiry in a secondary mathematics classroom. Cognitionand Instruction, 25 (2): 161-217 .
Stein, Catherine (2007). : Let’s Talk – Promoting Mathematical Discourse in the Classroom. Mathematics Teacher . November 2007, Volume 101, Issue 4:285-289 .
Stylianides (2007) Stylianides, A. J. (2007). Proof and proving in school mathematics. Journal for Research in Mathematics Education, 38 : 289-321.

Sullivan, P. (2012). The Australian Curriculum: Mathematics as an opportunity to support teachers and improve student learning. In B. Atweh, M. Goos, R. Jorgensen, & D. Siemon (Eds.), Engaging the Australian National Curriculum: Mathematics – Perspectives from the field (pp. 175–189). Retrieved from http://www.merga.net.au/sites/default/files/editor/books/1/Book.pdf

The Programme for International Student Assessment (PISA) The results of the 2015 assessment were published on 6th December 2016. https://www. oecd.org/pisa/aboutpisa/

TIMSS Jordan, 2015, National Center for Human Resources Development, Amman, Jordan

Theresa J. Grant ,T,Jane-Jane Lo, and Judi th Flowers (2007). Shaping Prospective Teachers' Justifications for Computation:

Challenges and Opportunities. Teaching Children Mathematics / September 2007: 112-116.

Thompson, D , Senk, S. ,and Johnson G (2012). Opportunities to Learn and Proof in High School Mathematics Textbooks. Journal for Research in Mathematics Education. 2012, Vol. 43, No. 3, 253–295.

Yackel, E. (2002). What we can learn from analyzing the teacher's role in collective argumentation. Journal of Mathematical Behavior, 21(4), 423-440. doi: 10.1016/S0732- 3123(02)00143-8

Wiggins, G. "'Get Real!'(2003): Assessing for Quantitative Literacy." In B. L. Madison and L. A. Steen (eds.), Quantitative Literacy: Why Numeracy Matters for Schools and Colleges. Princeton, N.J.: National Council on Education and the Disciplines.

Wolfe, J.(2010). "Rhetorical Numbers: A Case for Quantitative Writing in the Composition Classroom." College Composition and Communication, 2010, 61(3):452–475.

Citation:

Sabbagh S.A and Al-Khayat A.M (2019). The reasoning strategies used by mathematics teachers to promote students' thinking for the essential grades in Jordan: A mixed-methods study. Glob. Res. J. Educ. 7(3). Pp927-938 http://www.globalscienceresearchjournals.org/grje